



Cambridge International AS & A Level

CANDIDATE
NAME



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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2024

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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2



1 Solve the equation $8^{3-6x} = 4 \times 5^{-2x}$. Give your answer correct to 3 decimal places.

[4]

[illegible]

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2 Find the exact coordinates of the stationary point of the curve $y = e^{2x} \sin 2x$ for $0 \leq x \leq \frac{1}{2}\pi$. [5]

[illegible]



- 3 The square roots of $24 - 7i$ can be expressed in the Cartesian form $x + iy$, where x and y are real and exact.

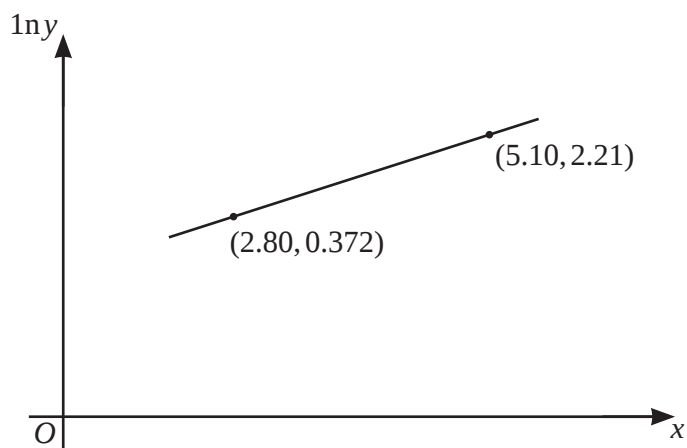
By first forming a quartic equation in x or y , find the square roots of $24 - 7i$ in exact Cartesian form.

[5]

[illegible]



4



The variables x and y satisfy the equation $ky = e^{cx}$, where k and c are constants. The graph of $\ln y$ against x is a straight line passing through the points $(2.80, 0.372)$ and $(5.10, 2.21)$, as shown in the diagram.

Find the values of k and c . Give each value correct to 2 significant figures. [4]

[illegible]



5 Express $\frac{6x^2 - 2x + 2}{(x-1)(2x+1)}$ in partial fractions. [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 6 (a)** On an Argand diagram shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 4 - 3i| \leq 2$ and $\arg(z - 2 - i) \geq \frac{1}{3}\pi$. [5]

- (b) Calculate the greatest value of $\arg z$ for points in this region. [2]

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7 Let $f(x) = 8x^3 + 54x^2 - 17x - 21$.

(a) Show that $x+7$ is a factor of $f(x)$. [1]

[illegible]

(b) Find the quotient when $f(x)$ is divided by $x + 7$. [2]

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(c) Hence solve the equation

$$8 \cos^3 \theta + 54 \cos^2 \theta - 17 \cos \theta - 21 = 0,$$

for $0^\circ \leq \theta \leq 360^\circ$.

[3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.



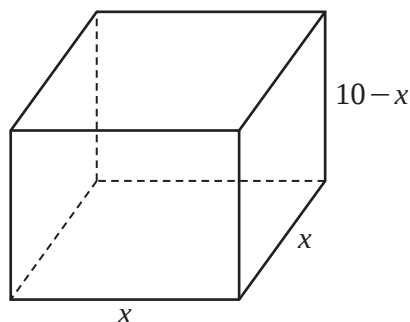
- 8 (a)** Express $3 \cos 2x - \sqrt{3} \sin 2x$ in the form $R \cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and α . [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



(b) Hence find the exact value of $\int_0^{\frac{1}{12}\pi} \frac{3}{(3 \cos 2x - \sqrt{3} \sin 2x)^2} dx$, simplifying your answer. [5]

[illegible]



A container in the shape of a cuboid has a square base of side x and a height of $(10 - x)$. It is given that x varies with time, t , where $t > 0$. The container decreases in volume at a rate which is inversely proportional to t .

When $t = \frac{1}{10}$, $x = \frac{1}{2}$ and the rate of decrease of x is $\frac{20}{37}$.

(a) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{-1}{2t(20x - 3x^2)}. \quad [5]$$

[illegible]



(b) Solve the differential equation, obtaining an expression for t in terms of x .

[6]

[illegible]



10 The equations of two straight lines are

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + 2a\mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j} + a\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -3\mathbf{i} - \mathbf{j} + 4\mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}),$$

where a is a constant.

- (a) Given that the acute angle between the directions of these lines is $\frac{1}{4}\pi$, find the possible values of a . [6]

[illegible]



- (b)** Given instead that the lines intersect, find the value of a and the position vector of the point of intersection. [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 11** Use the substitution $2x = \tan \theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{12}{(1+4x^2)^2} dx .$$

Give your answer in the form $a + b\pi$, where a and b are rational numbers. [9]

[illegible]

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Additional page

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